

**Remark 3.19.** *[005]* (Solved on 2022-10-11) Note that " $\forall x \in A, \varphi$ " is true if  $A$  is the empty set; this is consistent with what was discussed in the exercise *[016]*. This has though a striking consequence: the implication

$$(\forall x \in A, \varphi) \Rightarrow (\exists x \in A, \varphi)$$

is always valid when  $A$  is a non-empty set, but is instead false when  $A = \emptyset$ .