Exercises

E3.35 [016] As already commented in [00R], given A a set, and $P(x)$ a logical proposition dependent from a free variable x, we usally write

$$\forall x \in A, P(x) \quad , \quad \exists x \in A, P(x)$$

however

$$\forall x \in A, P(x) \text{ summarizes } \forall x, (x \in A) \Rightarrow P(x) \ ,$$

$$\exists x \in A, P(x) \text{ summarizes } \exists x, (x \in A) \wedge P(x) \ ;$$

where the "extended" versions are well-formed formulas.

Using this extended version you can prove that the two propositions

$$\neg(\forall x \in A, P(x)) \ , \ \exists x \in A, (\neg P(x)) \ .$$

are equivalent, in the sense that from one it is possible to prove the other (and vice versa). In the proof use only tautologies (listed in [00N]) and in particular the equivalence of the formula "$P \Rightarrow Q$" with "$(\neg P) \vee Q$" [a], and finally the equivalence between "$\neg\exists x, Q$" and "$\forall x, \neg Q$" [b].

Replacing $P(x)$ with $\neg P(x)$ and using the tautology of double negation finally results in

$$\forall x \in A, (\neg P(x)) \ , \ \neg(\exists x \in A, P(x))$$

are equivalent.

Solution 1. *[017]*

[a] Tautology in eqn. [(3.13)].

[b] Already discussed in eqn. [(3.16)].