E3.53 [01R] *(Solved on 2022-10-25)* The **axiom of good foundation** (also called **axiom of regularity**) of the Zermelo–Fraenkel theory says that every non-empty set X contains an element y that is disjoint from X; in formula

$$\forall X, X \neq \varnothing \Rightarrow (\exists y \, (y \in X) \land (X \cap y = \varnothing))$$

(remember that every object in the theory is a set, so y is a set). Using this axiom prove these facts.

- There is no set x that is an element of itself, that is, for which $x \in x$.

- More generally there is no finite family $x_1, \ldots x_n$ such that $x_1 \in x_2 \in \ldots \in x_n \in x_1$.

- There is also no $x_1, \ldots x_n, \ldots$ sequence of sets for which $x_1 \ni x_2 \ni x_3 \ni x_4 \ldots$.

Solution 1. [01S]

[[01T]] [[01V]]