

[026] The **axiom of union**^a says that for each set A there is a set B that contains all the elements of the elements of A ; in symbols,

$$\forall A \exists B, \forall x, (x \in B \iff (\exists y, y \in A \wedge x \in y)) .$$

This implies that this set is unique, by the axiom of extensionality [1Y8]; we indicate this set B with $\underline{\bigcup}A$ (so as not to confuse it with the symbol already introduced before).

For example if

$$A = \{\{1, 3, \{5, 2\}\}, \{7, 19\}\}$$

then

$$\underline{\bigcup}A = \{1, 3, \{5, 2\}, 7, 19\} .$$

Given A_1, \dots, A_k sets, let $D = \{A_1, \dots, A_k\}$ ^b we define

$$A_1 \cup A_2 \dots \cup A_k \stackrel{\text{def}}{=} \underline{\bigcup}D .$$

^aThis is the "official" version of Zermelo–Fraenkel. However, the simplified version [1Y2] is often used

^bThe existence of this set can be proven, see [029]