Exercises

E3.84 [02H]Prerequisites:[01M], [026],[020] . Let I be a non-empty set of indexes, let A_i a family of non-empty sets indexed by $i \in I$. Recall that, by definition, the Cartesian product $\prod_{i \in I} A_i$ is the set of functions $f : I \to \bigcup_{i \in I} A_i$ such that $f(i) \in A_i$ for each $i \in I$.

Show that the following are equivalent formulations of the **axiom of choice**.

- The Cartesian product of a non-empty family of non-empty sets is non-empty.
- Given a family A_i as above, such that the sets are not-empty and pairwise disjoint, there is a subset B of $\bigcup_{i \in I} A_i$ such that, for each $i \in I$, $B \cap A_i$ contains a single element.
- Let S be a set. Then there is a function $g : \mathcal{P}(S) \to S$ such that $g(A) \in A$ for each nonempty $A \in \mathcal{P}(S)$.

Solution 1. *[02J]*