Exercises

3.277 [05S] Now consider instead the characteristic function defined as before, but considered as $\mathbb{1}_A : X \to \mathbb{Z}_2$ i.e. taking values in the group $\mathbb{Z}_2$ (more correctly referred to as $\mathbb{Z}/2\mathbb{Z}$).

In this case the above relations can be written as

$$\mathbb{1}_{A^c} = \mathbb{1}_A + 1 \ , \ \mathbb{1}_{A\cap B} = \mathbb{1}_A \mathbb{1}_B \ , \ \mathbb{1}_{A\cup B} = \mathbb{1}_A \mathbb{1}_B + \mathbb{1}_A + \mathbb{1}_B \ .$$

Recall the definition of the symmetric difference $A\Delta B = (A \setminus B) \cup (B \setminus A)$, and then

$$\mathbb{1}_{A\Delta B} = \mathbb{1}_A + \mathbb{1}_B \ .$$

With these rules we show that

$$A\Delta B = B\Delta A \ , \ (A\Delta B)^c = A\Delta(B^c) = (A^c)\Delta B \ , \ A\Delta B = C \iff A$$

$$(A\Delta B) \cap C = (A \cap C)\Delta(B \cap C) \ , \ A \cup (B\Delta C) = (A \cup B)\Delta(A^c \cap C)$$