Exercises

E3.306 [05Z] We want to rewrite the tautologies seen in [00N] in the form of set relations.

Let X be a set and let $\alpha, \beta, \gamma \subseteq X$ be subsets. Let $x \in X$. If we define $A = (x \in \alpha)$, $B = (x \in \beta)$, $C = (x \in \gamma)$ in the tautologies, we can then rewrite each tautology as a formula between sets $\alpha, \beta, \gamma, X, \emptyset$, that use connectives $=, \cap, \cup$ and the complement.

Surprisingly, rewriting can be done algorithmically and in a purely syntactic manner. Pick a tautology seen in [00N]. In the following φ, ψ indicate subparts of tautology that are well-formed formulas.

- Replace $((\varphi) \Rightarrow (\psi))$ with $((\neg(\varphi)) \vee (\psi))$ (you will get another tautology).
- Then syntactically replace $\neg(\varphi)$ with $(\varphi)^c$, $\vee$ with $\cup$ and $\wedge$ with $\cap$; replace A with α, B with β, C with γ, V with X, and F with $\emptyset$.
- Finally, if the formula contains at least one ”$\iff$”, transform them all in ”=”; otherwise add ”$= X$” at the end.

Check that this ”algorithm” really works!