Definition 3.119. [06P] *Given a directed set* $(X, \leq_X)$ *a subset of it* $Y \subseteq X$ *is called* ***cofinal*** *if*

$$\forall x \in X \; \exists y \in Y, \; y \geq_X x \tag{3.120}$$

More in general, another directed set $(Z, \leq_Z)$ *is said to be* ***cofinal*** *in* X *if there exists a map* $i : Z \to X$ *monotonic weakly increasing and such that* $i(Z)$ *is is cofinal in* X*;* i.e.

$$(\forall z_1, z_2 \in Z, z_1 \leq_Z z_2 \Rightarrow i(z_1) \leq_X i(z_2)) \wedge (\forall x \in X \, \exists z \in Z, \; i(z) \geq_X x \tag{3.121}$$

(This second case generalizes the first one, where we may choose $i : Y \to X$ *to be the injection map, and* $\leq_Y$ *to be the restriction of* $\leq_X$ *to* Y*.)*