

## Exercises

3.219 [08F] Prerequisites: [01P], [07Z], [084], [080], [08C].

Let be given two well-ordered non-empty sets  $(X, \leq_X)$  and  $(Y, \leq_Y)$ . Show that

- (a) there is an initial segment  $S$  of  $X$  and a strictly increasing monotonic bijective function  $g : S \rightarrow Y$ ; or<sup>a</sup>
- (b) there is an initial segment  $T$  of  $Y$  and a bijective strictly increasing monotonic function  $g : X \rightarrow T$ .

In the first case we will write that  $(Y, \leq_Y) \preceq (X, \leq_X)$ , in the second that  $(X, \leq_X) \preceq (Y, \leq_Y)$ . (Note that in the first case you have  $|Y| \leq |X|$  and in the second  $|X| \leq |Y|$ ). By the previous exercise, the map  $g$  and its segment are unique.

**Solution 1.** [08G]

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<sup>a</sup>The two conditions can also both apply, in which case  $X, Y$  have the same type of order.