Theorem 4.b.2. [08Z]

Let A be a non-empty set; suppose that $a \in A$ is fixed, and functions $g_n : A \to A$ are given, one for each $n \in \mathbb{N}$. Then there exists an unique function $f : \mathbb{N} \to A$ such that

- $f(0) = a$, *and*
- *for every $n \in \mathbb{N}$ we have $f(S(n)) = g_n(f(n))$.*

We will say that the function f is ***defined by recurrence*** *by the two previous conditions.*

Proof. [090] □

More generally given $g_n : A^{n+1} \to A$, an unique function $f : \mathbb{N} \to A$ exists, such that $f(0) = a$ and for every $n \in \mathbb{N}$ $f(S(n)) = g_n(f(0), f(1), \dots f(n))$.