

Exercises

3.146 [092] Let D, C be non-empty sets and $f : D \rightarrow C$ a function. Let I be a non-empty family of indexes, $A_i \subseteq D$, for $i \in I$. Given $A \subseteq D$ remember that the **image** of A is the subset $f(A)$ of C given by

$$f(A) \stackrel{\text{def}}{=} \{f(x), x \in A\} .$$

Show these image properties.

$$\begin{aligned} f\left(\bigcup_{i \in I} A_i\right) &= \bigcup_{i \in I} f(A_i) \\ f\left(\bigcap_{i \in I} A_i\right) &\subseteq \bigcap_{i \in I} f(A_i) . \end{aligned}$$

Show that the function is injective if and only if

$$f(A_1 \cap A_2) = f(A_1) \cap f(A_2) \quad (3.146)$$

is an equality for every choice of $A_1, A_2 \subseteq D$.