

## Exercises

3.147 [093] Let  $A$  be a set and let  $g : A \rightarrow A$  be injective. We define the relation  $x \sim y$  which is true when an  $n \geq 0$  exists such that  $x = g^n(y)$  or  $x = g^{-n}(y)$ ; where

$$g^n = \overbrace{g \circ \cdots \circ g}^n$$

is the  $n$ -th iterate of the composition. (We decide that  $g^0$  is identity). Show that  $x \sim y$  is an equivalence relation. Study equivalence classes. Let  $U = \bigcap_{n=1}^{\infty} g^n(A)$  be the intersection of repeated images. Show that each class is entirely contained in  $U$  or is external to it.

**Solution 1.** [094]