Exercises

3.160 [09J] Let K be a positive integer, N an integer, and $I = \{N, N + 1, \ldots, N + K\}$ be the sequence of integers from N to $N + K$. For each $n \in I$ we set an integer values a_n. Let p be the only one polynomial of degree K such that $p(n) = a_n$ for every $n \in I$.

- Show that p has rational coefficients.
- Show that $p(x)$ is integer for every x integer.
- Find an example of a polynomial p which takes integer values for x integer, but not all coefficients of p are integers.
- What happens if I contains $K + 1$ integers, but not consecutive? Is it still true that, defining $p(x)$ as above, p only assumes integer values on integers?