

Exercises

E6.11 [OB0] Fix $I = \{1, \dots, n\}$. Let n distinct points $y_1, \dots, y_n \in \mathbb{R}$ be given; let $\sigma : I \rightarrow I$ be a permutation for which triangle inequalities between successive points are equalities *i.e.*

$$|y_{\sigma(i+2)} - y_{\sigma(i+1)}| + |y_{\sigma(i+1)} - y_{\sigma(i)}| = |y_{\sigma(i+2)} - y_{\sigma(i)}|$$

for $i = 1, \dots, n - 2$. Show that there are only two, we call them σ_1, σ_2 . Tip: Show that any such permutation necessarily puts the points "in order", *i.e.* you have

$$\forall i, y_{\sigma_1(i+1)} > y_{\sigma_1(i)} \quad , \quad \forall i, y_{\sigma_2(i+1)} < y_{\sigma_2(i)}$$

(up to deciding which is σ_1 and which is σ_2).

Solution 1. [OB1]