Proposition 6.36. *Let I be a set, $x_0 \in \mathbb{R}$ accumulation point for I, $f : I \to \mathbb{R}$ function, $l \in \mathbb{R}$.* [OBH]

Putting together all the definitions seen above, we get these definitions of limit.

In the case $x_0 \in \mathbb{R}$ and $l \in \mathbb{R}$:

$\lim_{x\to x_0} f(x) = l$	$\forall\varepsilon > 0, \exists\delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I \Rightarrow \lvert f(x) - l\rvert < \varepsilon$
$\lim_{x\to x_0^+} f(x) = l$	$\forall\varepsilon > 0, \exists\delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I \Rightarrow \lvert f(x) - l\rvert < \varepsilon$
$\lim_{x\to x_0^-} f(x) = l$	$\forall\varepsilon > 0, \exists\delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I \Rightarrow \lvert f(x) - l\rvert < \varepsilon$

Be $x_0 \in \mathbb{R}$, $l = \pm\infty$.

$\lim_{x\to x_0} f(x) = \infty$	$\forall z, \exists\delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I \Rightarrow f(x) > z$
$\lim_{x\to x_0} f(x) = -\infty$	$\forall z, \exists\delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I \Rightarrow f(x) < z$
$\lim_{x\to x_0^+} f(x) = \infty$	$\forall z, \exists\delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I \Rightarrow f(x) > z$
$\lim_{x\to x_0^+} f(x) = -\infty$	$\forall z, \exists\delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I \Rightarrow f(x) < z$
$\lim_{x\to x_0^-} f(x) = \infty$	$\forall z, \exists\delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I \Rightarrow f(x) > z$
$\lim_{x\to x_0^-} f(x) = -\infty$	$\forall z, \exists\delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I \Rightarrow f(x) < z$

Let $l \in \mathbb{R}$, $x_0 = \pm\infty$.

$\lim_{x\to\infty} f(x) = l$	$\forall\varepsilon > 0, \exists y, \forall x, x > y, x \in I \Rightarrow \lvert f(x) - l\rvert < \varepsilon$
$\lim_{x\to-\infty} f(x) = l$	$\forall\varepsilon > 0, \exists y, \forall x, x < y, x \in I \Rightarrow \lvert f(x) - l\rvert < \varepsilon$
$\lim_{x\to\infty} f(x) = \infty$	$\forall z, \exists y, \forall x, x > y, x \in I \Rightarrow f(x) > z$
$\lim_{x\to-\infty} f(x) = \infty$	$\forall z, \exists y, \forall x, x < y, x \in I \Rightarrow f(x) > z$
$\lim_{x\to\infty} f(x) = -\infty$	$\forall z, \exists y, \forall x, x > y, x \in I \Rightarrow f(x) < z$
$\lim_{x\to-\infty} f(x) = -\infty$	$\forall z, \exists y, \forall x, x < y, x \in I \Rightarrow f(x) < z$