Proposition 6.43. *In the case $x_0 \in \mathbb{R}$ and $l \in \mathbb{R}$, we divide the definition into two conditions:* [46] [OBK]

$\limsup_{x\to x_0} f(x) \le l$ $\limsup_{x\to x_0} f(x) \ge l$	$\forall \varepsilon > 0, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I \Rightarrow f(x) < l + \varepsilon$ $\forall \varepsilon > 0, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I, f(x) > l - \varepsilon$
$\limsup_{x\to x_0^+} f(x) \le l$ $\limsup_{x\to x_0^+} f(x) \ge l$	$\forall \varepsilon > 0, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I \Rightarrow f(x) < l + \varepsilon$ $\forall \varepsilon > 0, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I, f(x) > l - \varepsilon$
$\limsup_{x\to x_0^-} f(x) \le l$ $\limsup_{x\to x_0^-} f(x) \ge l$	$\forall \varepsilon > 0, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I \Rightarrow f(x) < l + \varepsilon$ $\forall \varepsilon > 0, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I, f(x) > l - \varepsilon$

$\liminf_{x\to x_0} f(x) \ge l$ $\liminf_{x\to x_0} f(x) \le l$	$\forall \varepsilon > 0, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I \Rightarrow f(x) > l - \varepsilon$ $\forall \varepsilon > 0, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I, f(x) < l + \varepsilon$
$\liminf_{x\to x_0^+} f(x) \ge l$ $\liminf_{x\to x_0^+} f(x) \le l$	$\forall \varepsilon > 0, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I \Rightarrow f(x) > l - \varepsilon$ $\forall \varepsilon > 0, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I, f(x) < l + \varepsilon$
$\liminf_{x\to x_0^-} f(x) \ge l$ $\liminf_{x\to x_0^-} f(x) \le l$	$\forall \varepsilon > 0, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I \Rightarrow f(x) > l - \varepsilon$ $\forall \varepsilon > 0, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I, f(x) < l + \varepsilon$

In the case $x_0 \in \mathbb{R}$ and $l = \pm\infty$:

$\limsup_{x\to x_0} f(x) = \infty$	$\forall z, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I, f(x) > z$
$\limsup_{x\to x_0^+} f(x) = \infty$	$\forall z, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I, f(x) > z$
$\limsup_{x\to x_0^-} f(x) = \infty$	$\forall z, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I, f(x) > z$
$\limsup_{x\to x_0} f(x) = -\infty$	$\forall z, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I \Rightarrow f(x) < z$
$\limsup_{x\to x_0^+} f(x) = -\infty$	$\forall z, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I \Rightarrow f(x) < z$
$\limsup_{x\to x_0^-} f(x) = -\infty$	$\forall z, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I \Rightarrow f(x) < z$
$\liminf_{x\to x_0} f(x) = \infty$	$\forall z, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I \Rightarrow f(x) > z$
$\liminf_{x\to x_0^+} f(x) = \infty$	$\forall z, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I \Rightarrow f(x) > z$
$\liminf_{x\to x_0^-} f(x) = \infty$	$\forall z, \exists \delta > 0, \forall x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I \Rightarrow f(x) > z$
$\liminf_{x\to x_0} f(x) = -\infty$	$\forall z, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x \neq x_0, x \in I, f(x) < z$
$\liminf_{x\to x_0^+} f(x) = -\infty$	$\forall z, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x > x_0, x \in I, f(x) < z$
$\liminf_{x\to x_0^-} f(x) = -\infty$	$\forall z, \forall \delta > 0, \exists x, \lvert x - x_0\rvert < \delta, x < x_0, x \in I, f(x) < z$

In the case $x_0 = \pm\infty$ and $l = \pm\infty$:

$\limsup_{x\to\infty} f(x) = \infty$	$\forall z, \forall y, \exists x, x > y, x \in I, f(x) > z$
$\limsup_{x\to-\infty} f(x) = \infty$	$\forall z, \forall y, \exists x, x < y, x \in I, f(x) > z$
$\limsup_{x\to\infty} f(x) = -\infty$	$\forall z, \exists y, \forall x, x > y, x \in I \Rightarrow f(x) < z$
$\limsup_{x\to-\infty} f(x) = -\infty$	$\forall z, \exists y, \forall x, x < y, x \in I \Rightarrow f(x) < z$
$\liminf_{x\to\infty} f(x) = \infty$	$\forall z, \exists y, \forall x, x > y, x \in I \Rightarrow f(x) > z$
$\liminf_{x\to-\infty} f(x) = \infty$	$\forall z, \exists y, \forall x, x < y, x \in I \Rightarrow f(x) > z$
$\liminf_{x\to\infty} f(x) = -\infty$	$\forall z, \forall y, \exists x, x > y, x \in I, f(x) < z$
$\liminf_{x\to-\infty} f(x) = -\infty$	$\forall z, \forall y, \exists x, x < y, x \in I, f(x) < z$

In the case $x_0 = \pm\infty$ and $l \in \mathbb{R}$:

$\limsup_{x\to\infty} f(x) \le l$ $\limsup_{x\to\infty} f(x) \ge l$	$\forall \varepsilon > 0, \exists y, \forall x, x > y, x \in I \Rightarrow f(x) < l + \varepsilon$ $\forall \varepsilon > 0, \forall y, \exists x, x > y, x \in I, f(x) > l - \varepsilon$
$\limsup_{x\to-\infty} f(x) \le l$ $\limsup_{x\to-\infty} f(x) \ge l$	$\forall \varepsilon > 0, \exists y, \forall x, x < y, x \in I \Rightarrow f(x) < l + \varepsilon$ $\forall \varepsilon > 0, \forall y, \exists x, x < y, x \in I, f(x) > l - \varepsilon$
$\liminf_{x\to\infty} f(x) \le l$ $\liminf_{x\to\infty} f(x) \ge l$	$\forall \varepsilon > 0, \forall y, \exists x, x > y, x \in I, f(x) < l + \varepsilon$ $\forall \varepsilon > 0, \exists y, \forall x, x > y, x \in I \Rightarrow f(x) > l - \varepsilon$
$\liminf_{x\to-\infty} f(x) \le l$ $\liminf_{x\to-\infty} f(x) \ge l$	$\forall \varepsilon > 0, \forall y, \exists x, x < y, x \in I, f(x) < l + \varepsilon$ $\forall \varepsilon > 0, \exists y, \forall x, x < y, x \in I \Rightarrow f(x) > l - \varepsilon$

[46] In the following tables all commas "," after the last quantifier should be interpreted as conjunctions "$\wedge$", but were written as "," for lighten the notation.