Proposizione 6.43. *Nel caso $x_0 \in \mathbb{R}$ e $l \in \mathbb{R}$, dividiamo la definizione in due condizioni:* [46] [OBK]

$\limsup_{x\to x_0} f(x) \le l$ $\limsup_{x\to x_0} f(x) \ge l$	$\forall\varepsilon>0,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x\ne x_0,x\in I\Rightarrow f(x)<l+\varepsilon$ $\forall\varepsilon>0,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x\ne x_0,x\in I,f(x)>l-\varepsilon$
$\limsup_{x\to x_0^+} f(x) \le l$ $\limsup_{x\to x_0^+} f(x) \ge l$	$\forall\varepsilon>0,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x>x_0,x\in I\Rightarrow f(x)<l+\varepsilon$ $\forall\varepsilon>0,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x>x_0,x\in I,f(x)>l-\varepsilon$
$\limsup_{x\to x_0^-} f(x) \le l$ $\limsup_{x\to x_0^-} f(x) \ge l$	$\forall\varepsilon>0,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x<x_0,x\in I\Rightarrow f(x)<l+\varepsilon$ $\forall\varepsilon>0,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x<x_0,x\in I,f(x)>l-\varepsilon$

$\liminf_{x\to x_0} f(x) \ge l$ $\liminf_{x\to x_0} f(x) \le l$	$\forall\varepsilon>0,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x\ne x_0,x\in I\Rightarrow f(x)>l-\varepsilon$ $\forall\varepsilon>0,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x\ne x_0,x\in I,f(x)<l+\varepsilon$
$\liminf_{x\to x_0^+} f(x) \ge l$ $\liminf_{x\to x_0^+} f(x) \le l$	$\forall\varepsilon>0,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x>x_0,x\in I\Rightarrow f(x)>l-\varepsilon$ $\forall\varepsilon>0,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x>x_0,x\in I,f(x)<l+\varepsilon$
$\liminf_{x\to x_0^-} f(x) \ge l$ $\liminf_{x\to x_0^-} f(x) \le l$	$\forall\varepsilon>0,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x<x_0,x\in I\Rightarrow f(x)>l-\varepsilon$ $\forall\varepsilon>0,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x<x_0,x\in I,f(x)<l+\varepsilon$

Nel caso $x_0 \in \mathbb{R}$ e $l = \pm\infty$:

$\limsup_{x\to x_0} f(x) = \infty$	$\forall z,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x\ne x_0,x\in I,f(x)>z$
$\limsup_{x\to x_0^+} f(x) = \infty$	$\forall z,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x>x_0,x\in I,f(x)>z$
$\limsup_{x\to x_0^-} f(x) = \infty$	$\forall z,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x<x_0,x\in I,f(x)>z$
$\limsup_{x\to x_0} f(x) = -\infty$	$\forall z,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x\ne x_0,x\in I\Rightarrow f(x)<z$
$\limsup_{x\to x_0^+} f(x) = -\infty$	$\forall z,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x>x_0,x\in I\Rightarrow f(x)<z$
$\limsup_{x\to x_0^-} f(x) = -\infty$	$\forall z,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x<x_0,x\in I\Rightarrow f(x)<z$

$\liminf_{x\to x_0} f(x) = \infty$	$\forall z,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x\ne x_0,x\in I\Rightarrow f(x)>z$
$\liminf_{x\to x_0^+} f(x) = \infty$	$\forall z,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x>x_0,x\in I\Rightarrow f(x)>z$
$\liminf_{x\to x_0^-} f(x) = \infty$	$\forall z,\exists\delta>0,\forall x,\lvert x-x_0\rvert<\delta,x<x_0,x\in I\Rightarrow f(x)>z$
$\liminf_{x\to x_0} f(x) = -\infty$	$\forall z,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x\ne x_0,x\in I,f(x)<z$
$\liminf_{x\to x_0^+} f(x) = -\infty$	$\forall z,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x>x_0,x\in I,f(x)<z$
$\liminf_{x\to x_0^-} f(x) = -\infty$	$\forall z,\forall\delta>0,\exists x,\lvert x-x_0\rvert<\delta,x<x_0,x\in I,f(x)<z$

Nel caso $x_0 = \pm\infty$ e $l = \pm\infty$:

$\limsup_{x\to\infty} f(x) = \infty$	$\forall z,\forall y,\exists x,x>y,x\in I,f(x)>z$
$\limsup_{x\to-\infty} f(x) = \infty$	$\forall z,\forall y,\exists x,x<y,x\in I,f(x)>z$
$\limsup_{x\to\infty} f(x) = -\infty$	$\forall z,\exists y,\forall x,x>y,x\in I\Rightarrow f(x)<z$
$\limsup_{x\to-\infty} f(x) = -\infty$	$\forall z,\exists y,\forall x,x<y,x\in I\Rightarrow f(x)<z$

$\liminf_{x\to\infty} f(x) = \infty$	$\forall z,\exists y,\forall x,x>y,x\in I\Rightarrow f(x)>z$
$\liminf_{x\to-\infty} f(x) = \infty$	$\forall z,\exists y,\forall x,x<y,x\in I\Rightarrow f(x)>z$
$\liminf_{x\to\infty} f(x) = -\infty$	$\forall z,\forall y,\exists x,x>y,x\in I,f(x)<z$
$\liminf_{x\to-\infty} f(x) = -\infty$	$\forall z,\forall y,\exists x,x<y,x\in I,f(x)<z$

Nel caso $x_0 = \pm\infty$ e $l \in \mathbb{R}$:

$\limsup_{x\to\infty} f(x) \le l$ $\limsup_{x\to\infty} f(x) \ge l$	$\forall\varepsilon>0,\exists y,\forall x,x>y,x\in I\Rightarrow f(x)<l+\varepsilon$ $\forall\varepsilon>0,\forall y,\exists x,x>y,x\in I,f(x)>l-\varepsilon$
$\limsup_{x\to-\infty} f(x) \le l$ $\limsup_{x\to-\infty} f(x) \ge l$	$\forall\varepsilon>0,\exists y,\forall x,x<y,x\in I\Rightarrow f(x)<l+\varepsilon$ $\forall\varepsilon>0,\forall y,\exists x,x<y,x\in I,f(x)>l-\varepsilon$

$\liminf_{x\to\infty} f(x) \le l$ $\liminf_{x\to\infty} f(x) \ge l$	$\forall\varepsilon>0,\forall y,\exists x,x>y,x\in I,f(x)<l+\varepsilon$ $\forall\varepsilon>0,\exists y,\forall x,x>y,x\in I\Rightarrow f(x)>l-\varepsilon$
$\liminf_{x\to-\infty} f(x) \le l$ $\liminf_{x\to-\infty} f(x) \ge l$	$\forall\varepsilon>0,\forall y,\exists x,x<y,x\in I,f(x)<l+\varepsilon$ $\forall\varepsilon>0,\exists y,\forall x,x<y,x\in I\Rightarrow f(x)>l-\varepsilon$

[46] Nelle seguenti tabelle tutte le virgole "," dopo l'ultimo quantificatore devono essere interpretate come congiunzioni "∧", ma sono state scritte come "," per alleggerire la notazione.