

Exercise 6.45. [OBP] Let $A_1, A_2 \dots$ be sets, for $n \in \mathbb{N}$; let $X = \bigcup_n A_n$. We define the characteristic function $\mathbb{1}_A : X \rightarrow \mathbb{R}$ as

$$\mathbb{1}_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases} .$$

We will use the definitions $\limsup_n A_n$ and $\liminf_n A_n$ seen in eqn. [(3.286)] and [(3.287)]. You have

$$\mathbb{1}_{(\limsup_n A_n)} = \limsup_n \mathbb{1}_{A_n} , \quad (6.46)$$

$$\mathbb{1}_{(\liminf_n A_n)} = \liminf_n \mathbb{1}_{A_n} . \quad (6.47)$$