

Exercise 6.47. *[OBQ]* We fix a real valued sequence a_n . Now consider the definition of *[20F]* setting $I = \mathbb{N}$ and $x_0 = \infty$, so that neighborhoods of x_0 are sets containing $[n, \infty) = \{m \in \mathbb{N} : m \geq n\}$; with these assumptions show that you have

$$\limsup_{n \rightarrow \infty} a_n = \inf_n \sup_{m \geq n} a_n = \lim_{n \rightarrow \infty} \sup_{m \geq n} a_n ,$$

$$\liminf_{n \rightarrow \infty} a_n = \sup_n \inf_{m \geq n} a_n = \lim_{n \rightarrow \infty} \inf_{m \geq n} a_n ., \quad (6.48)$$