Exercises

E6.52 [OBW] Prerequisites: [OBS]. Given $x \in \mathbb{R}$ and $N \in \mathbb{N}, N \geq 2$, prove that at least one element of the set $\{x, 2x, \ldots, (N-1)x\}$ is at most distance $1/N$ from an integer, that is, there exist $n, m \in \mathbb{Z}$ with $1 \leq n \leq N-1$ such that $|nx - m| \leq 1/N$.

Solution 1. *[OBX]*