Exercises

E6.52 Prerequisites: [OBS], [OBW]. Given $x, b \in \mathbb{R}$ with $x \neq 0$ irrational, and $\varepsilon > 0$, prove that there is a natural M such that $Mx - b$ is at most ε from an integer. [OBY]

Let $\varphi(x) = x - \lfloor x \rfloor$ be the fractional part of x, we have $\varphi(x) \in [0, 1)$. The above result implies that the sequence $\varphi(nx)$ is dense in the interval $[0, 1]$.

Note that instead if $x \neq 0$ is rational *i.e.* $x = n/d$ with n, d coprime integers and $d > 0$, then the sequence $\varphi(nx)$ assumes all and only the values $\{0, 1/d, 2/d, \dots (d-1)/d\}$.

(This is demonstrated by the Bézout's lemma [26]).

Solution 1. *[OBZ]*

[[OCO]]