Exercises

E6.52 [OC1]Prerequisites:[OBW]. *(Dirichlet's approximation theorem)* Given an irrational number x, show that there are infinitely many rationals α such that we can represent $\alpha = m/n$ in order to satisfy the relation

$$\left|x - \frac{m}{n}\right| < \frac{1}{n^2} \quad .$$

Some comments.

- Note for every fixed $n \geq 2$ there is at most an m for which the previous relation holds; but there may not be one.
- Note that if the relation holds for a rational α, there are only finite choices of representations for which it holds,
- and certainly it holds for the "canonical" representation with n, m coprimes.

Solution 1. *[OC2]*

[2B0]