E6.52 [OC5] Prove that for every rational m/n you have

$$\left| \sqrt{2} - \frac{m}{n} \right| > \frac{1}{4n^2}.$$

We obtain that the set $A = \bigcup_{m \in \mathbb{Z}, n \in \mathbb{N}^*} \left(\frac{m}{n} - \frac{1}{4n^2}, \frac{m}{n} + \frac{1}{4n^2} \right)$ is an open set that contains every rational number, but $A \neq \mathbb{R}$.

Solution 1. [OC6]