

Exercises

E6.60 [OCF] Difficulty:*. More generally, given $p(x) = a_0 + a_1x + \cdots + a_nx^n$, $p \in \mathbb{Q}[z]$ $q(x) = b_0 + b_1x + \cdots + b_mx^m$, $q \in \mathbb{Q}[z]$, and given α, β such that $p(\alpha) = 0 = q(\beta)$, construct a polynomial $r \in \mathbb{Q}[z]$ such that $r(\alpha + \beta) = 0$.

(Hint. use the theory of the resultant [35]).

So if α, β are algebraic then $\alpha + \beta$ is algebraic.

Solution 1. [OCG]