

## 7 Sequences and series

[OCN]

### 7.1 Sequences

Let  $(a_n)_{n \in \mathbb{N}} \subseteq \mathbb{R}$  be a real-valued sequence (as defined in [16G]).

Given  $N \in \mathbb{N}$  we will write  $\sup_{n \geq N} a_n$  in the following, instead of  $\sup\{a_N, a_{N+1} \dots\}$ , and similarly for the infimum. (This is in accordance with [20H])

#### Exercises

E7.1 [OCF]

E7.2 [OCR]

E7.3 [OCS]

E7.4 [OCV]

E7.5 [OCX]

E7.6 [ODO]

E7.7 [OD2]

E7.8 [OD4]

E7.9 [OD6]

E7.10 [OD9]

E7.11 [ODD]

**Exercise 7.12.** [ODJ]**Exercise 7.13.** [O2F]**QuasiEsercizio 21.** [ODG]

See also exercises [OB9] and [OB7].

#### 7.1.1 Summation by parts

##### Exercises

E7.14 [217]

E7.15 [21H]

**QuasiEsercizio 22.** [ODF]

### 7.2 Recursive sequences

#### Exercises

E7.16 [ODK]

E7.17 [ODN]

## 7.3 Series

### 7.3.1 Tests

**Theorem 7.18** (Root test). [219]

**Theorem 7.19** (Ratio test). [21C]

**Remark 7.20.** [0F1]

**Theorem 7.21.** [21D]

The Dirichlet criteria implies the Leibniz “alternating series test” criteria.

**Theorem 7.22** (Dirichlet criterion). [21F]

In particular, if we set  $a_n = (-1)^n$  we prove the existence of the limit in Leibniz test.

**Theorem 7.23** (Alternating series test, or Leibniz test). [238]

**Theorem 7.24.** [ODR]

### 7.3.2 Exercises

#### Exercises

E7.25 [214]

E7.26 [23D]

E7.27 [ODW]

E7.28 [ODY]

E7.29 [0F0]

E7.30 [0F2]

E7.31 [0F4]

E7.32 [0F5]

E7.33 [0F7]

E7.34 [0F8]

E7.35 [0F9]

E7.36 [21M]

E7.37 [23F]

E7.38 [20Z]

E7.39 [210]

**QuasiEsercizio 23.** [ODT]

**QuasiEsercizio 24.** [ODV]

**QuasiEsercizio 25.** [0FC]

**QuasiEsercizio 26.** [0FD]

**QuasiEsercizio 27.** [0FF]

**QuasiEsercizio 28.** [0FG]

**QuasiEsercizio 29.** [ODH]

See also exercise [1T9].

### 7.3.3 Cauchy product

**Definition 7.40.** [OFH]

#### Exercises

E7.41 [OFJ]

E7.42 [OFK]

E7.43 [OFM]

E7.44 [OFP]

See also exercise [1KQ].

## 7.4 Generalized sequences, or “nets”

[29X]

## 7.5 Generalized series

### 7.5.1 Generalized series with positive terms

**Definition 7.45.** [OFW]

#### Exercises

E7.46 [OFX]

E7.47 [OFZ]

E7.48 [OG0]

E7.49 [OG3]

[ [OG4] ]