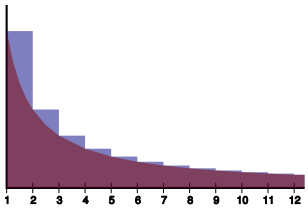

Figure 1: Representation of Euler-Mascheroni constant
Image by William Demchick, Creative Commons Attribution 3.0 Unported License, taken from wikipedia.

Exercises

E7.i.13 `Topics:Euler-Mascheroni constant.Prerequisites:[211].` [OD6]

Show that the limit

$$\gamma = \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k} - \log(n)\right) \quad .$$

exists and is finite. This γ is called Costante di Eulero - Mascheroni. It can be defined in many different ways (see the previous link) including

$$\gamma = \int_1^{\infty}\left(\frac{1}{\lfloor x\rfloor} - \frac{1}{x}\right)\mathbb{d}x$$

where the parentheses $\lfloor\cdot\rfloor$ indicate the floor function $\lfloor x\rfloor \stackrel{\text{def}}{=} \max\{n\in\mathbb{Z} : n\leq x\}$. In the image 1 the constant γ is the blue area.

Solution 1. *[OD8]*