

Exercises

E7.4 [ODD] *Note: Exercise 1 from the written exam 9 April 2011.* Let (a_n) be a sequence of real numbers, with $a_n \geq 0$.

(a) Show that if $\sum_{n=1}^{\infty} a_n$ converges then also

$$\sum_{n=1}^{\infty} a_n^2 \quad \text{e} \quad \sum_{n=1}^{\infty} \left(a_n \sum_{m=n+1}^{\infty} a_m \right)$$

converge.

(b) Assuming moreover that $\sum_{n=1}^{\infty} a_n$ is convergent, let's define

$$a = \sum_{n=1}^{\infty} a_n \quad , \quad b = \sum_{n=1}^{\infty} \left(a_n \sum_{m=n+1}^{\infty} a_m \right) \quad , \quad c = \sum_{n=1}^{\infty} a_n^2$$

then show that $a^2 = 2b + c$.