Exercise 7.13. [0DJ]*Let a_n, b_n be real sequences (which can have variable signs, take value zero, and are not necessarily infinitesimal).*
Recall that the notation $a_n = o(b_n)$ means:

$$\forall \varepsilon > 0,\ \exists \overline{n} \in \mathbb{N},\ \forall n \in \mathbb{N}, n \geq \overline{n} \Rightarrow |a_n| \leq \varepsilon |b_n| \ .$$

Shown that these two clauses are equivalent.

- *Eventually in n we have that $a_n = 0 \iff b_n = 0$; having specified this, we have $\lim_n \frac{a_n}{b_n} = 1$, where it is decided that $0/0 = 1$ (in particular a_n, b_n eventually have the same sign, when they are not both null);*
- *we have that $a_n = b_n + o(b_n)$.*

The second condition appears in Definition 3.2.7 in [3] where it is indicated by the notation $a_n \sim b_n$.
Deduct that $a_n \sim b_n$ is an equivalence relation.

Solution 1. [29Y]