Theorem 7.27. [ODR](Solved on 2022-12-13) *Consider the series $\sum_{n=1}^{\infty} a_n$ where the terms are positive: $a_n > 0$. Define*

$$z_n = n\left(\frac{a_n}{a_{n+1}} - 1\right)$$

for convenience.

- *If $z_n \leq 1$ eventually in n, then the series does not converge.*
- *If there exists $L > 1$ such that $z_n \geq L$ eventually in n,* i.e. *equivalently if*

  $$\liminf_{n\to\infty} z_n > 1 \quad ,$$

  then the series converges.

In addition, fixed $h \in \mathbb{Z}$, we can define

$$z_n = (n+h)\left(\frac{a_n}{a_{n+1}} - 1\right)$$

or

$$z_n = n\left(\frac{a_{n+h}}{a_{n+h+1}} - 1\right)$$

such as

$$z_n = n\left(\frac{a_{n-1}}{a_n} - 1\right)$$

and the criterion applies in the same way.

Solution 1. [ODS]