Exercises

E7.31 [0F4] Note: *Written exam of 4th Apr 2009, exee 1.(Proposed on 2022-12-13)* Given a sequence $(a_n)_n$ of strictly positive numbers, it is said that *the infinite product* $\prod_{n=0}^{\infty} a_n$ *converges* if there exists finite and strictly positive the limit of partial products, i.e.

$$\lim_{N\to+\infty} \prod_{n=0}^{N} a_n \in (0,+\infty) \quad .$$

Prove that

(a) if $\prod_{n=0}^{\infty} a_n$ converges then $\lim_{n\to+\infty} a_n = 1$;

(b) if the series $\sum_{n=0}^{\infty} |a_n - 1|$ converges, then it also converges $\prod_{n=0}^{\infty} a_n$;

(c) find an example where the series $\sum_{n=0}^{\infty}(a_n - 1)$ converges but $\prod_{n=0}^{\infty} a_n = 0$.