

Exercises

E7.31 [OF5] We indicate with $\mathcal{P}_f(\mathbb{N})$ the set of subsets $B \subseteq \mathbb{N}$ which are finite sets. This is said *the set of finite parts*.

We abbreviate $\mathcal{P} = \mathcal{P}_f(\mathbb{N})$ in the following.

Given a sequence $(a_n)_n$ of real numbers and a $B \in \mathcal{P}$ we indicate with $s(B) = \sum_{n \in B} a_n$ the finite sum with indices in B .

Suppose the series $\sum_{n=0}^{\infty} a_n$ converge but not converge at all. Then:

- $\{s(F) : F \in \mathcal{P}\}$ it is dense in \mathbb{R} .
- There is a reordering σ of \mathbb{N} , that is, a bijective function $\sigma : \mathbb{N} \rightarrow \mathbb{N}$, such that all partial sums $\sum_{n=0}^N a_{\sigma(n)}$ (at the variation of N) is dense in \mathbb{R} .

[[OF6]]