Exercises

E7.31 [OF7] Note: *This result is attributed to Riemann , see 3.54 in [16].*.

Let be given a sequence $(a_n)_n$ of real numbers such that $\sum_{n=0}^{\infty} a_n$ converges (to a finite value) but $\sum_{n=0}^{\infty} |a_n| = \infty$; for each l, L with $-\infty \leq l \leq L \leq +\infty$ there is a permutation $\pi : \mathbb{N} \to \mathbb{N}$ such that, defining $S_N = \sum_{k=0}^{N} a_{\pi(k)}$, we have that

$$\limsup_{N\to\infty} S_N = L \quad , \quad \liminf_{N\to\infty} S_N = l \quad .$$