Exercises

E7.52 [OFT] Suppose f is monotonic, show that $\lim_{j\in J} f(j)$ exists (possibly infinite) and coincides with $\sup_J f$ (if it is increasing) or with $\inf_J f$ (if it is decreasing).

Infer that

$$\limsup_{j\in J} f(j) \stackrel{\text{def}}{=} \lim_{j\in J} \sup_{k\geq j} f(k)$$

$$\liminf_{j\in J} f(j) \stackrel{\text{def}}{=} \lim_{j\in J} \inf_{k\geq j} f(k)$$

are always well defined.