Exercises

E8.5 [OGS] Note: *Written exam of 25 March 2017.* Let (X, τ), (Y, θ) be two topological spaces with non-empty intersection and assume that the topologies restricted to $C = X \cap Y$ coincide (i.e. $\tau_{|C} = \theta_{|C}$) [a] and that C is open in both topologies (i.e. $C \in \tau, C \in \theta$). Prove that there is only one topology σ on $Z = X \cup Y$ such that $\sigma_{|X} = \tau$ and $\sigma_{|Y} = \theta$ and that $X, Y \in \sigma$.

Solution 1. *[OGT]*

[a] Remember that $\tau_{|C} = \{B \cap C : B \in \tau\}$.