

Exercises

E8.a.10 [OH7] Topics:boundary. Let $A \subset X$. Let's remember the definition of *boundary* $\partial A = \overline{A} \setminus A^\circ$. Note that ∂A is closed: indeed setting $B = A^c$ to be the complement, it is easily verified that $\partial A = \overline{A} \cap \overline{B}$. In particular we showed that $\partial A = \partial B$.

Show that the three sets $\partial A, A^\circ, B^\circ$ are disjoint, and that their union is X ; in particular, show that the three sets are characterized by these three properties:

- Each neighborhood of x intersects both A and B ;
- there exists a neighborhood x contained in A ;
- there exists a neighborhood x contained in B .

(See also [OQ3] for the case of metric spaces).

Solution 1. [OH8]