

Exercises

E8.33 [OHP] Prerequisites: [OKX], [OKZ]. Let $X = \mathbb{R} \cup \{+\infty, -\infty\}$, consider the family \mathcal{B} of parts of X that contains

- open intervals (a, b) with $a, b \in \mathbb{R}$ and $a < b$,
- half-lines $(a, +\infty] = (a, +\infty) \cup \{+\infty\}$ with $a \in \mathbb{R}$,
- the half-lines $[-\infty, b) = (-\infty, b) \cup \{-\infty\}$ with $b \in \mathbb{R}$.

(Note the similarity of sets in the second and third points with the "neighbourhoods of infinity" seen in Sec. [29H]).

Show that \mathcal{B} satisfies the properties (a), (b) seen in [OKX]. Let τ therefore be the topology generated from this base. The topological space (X, τ) is called **extended line**, often denoted $\overline{\mathbb{R}}$.

This topological space is T_2 , it is compact (Exercise [OJB]), and is homeomorphic to the interval $[0, 1]$. It can be equipped with a distance that generates the topology described above.

Solution 1. [OHQ]