Exercises

E8.34 [OHR] Prerequisites:[OKX],[OKZ].Let $X = \mathbb{R} \cup \{\infty\}$, let's consider the family $\mathcal{B}$ of parts of X comprised of

- the open intervals (a, b) with $a, b \in \mathbb{R}$ and $a < b$,
- the sets $(a, +\infty) \cup (-\infty, b) \cup \{\infty\}$ with $a, b \in \mathbb{R}$ and $a < b$.

Show that $\mathcal{B}$ satisfies the properties *(a),(b)* seen in [OKX]. Let τ therefore be the topology generated by this base. The topological space (X, τ) is called **one-point compactified line**. This topological space is T_2 and it is compact (Exer. [OJD]); it is homeomorphic to the circle (Exer. [OYF]); therefore it can be equipped with a distance that generates the topology described above.