Exercises

E8.c.2 [0J1] Prerequisites: [252]. Let X be a set and $\mathcal{V} \subseteq \mathcal{P}(X)$ a family of parts of X; we define τ as the intersection of all topologies that contain $\mathcal{V}$ *i.e.*

$$\tau \overset{\text{def}}{=} \bigcap \{\sigma, \sigma \supseteq \mathcal{V}, \sigma \text{ topology in } X\}$$

Show that τ is a topology.

τ is the *"topology generated by $\mathcal{V}$"*; it is also called *"the smallest topology that contains $\mathcal{V}$"*.