Exercises

E9.1.9 [OJN] An *ear* of a polygon is the triangle *ABC* formed by three consecutive vertices *A*, *B*, *C* of the polygon, such that the segment *AC* lies inside the polygon. This implies that the triangle *ABC* does not contain any point of the polygonal curve in its interior; and that the two segments *AB*, *BC* can be removed from the polygon and replaced with *AC* to create a newer polygon. Two ears are *non-overlapping* if their interiors do not intersect, or equivalently if the do not have a side in common.

Prove the Two ears theorem: every polygon with more than three vertices has at least two non-overlapping ears. (See [**?**, **?**] for more details).

(Hint: consider labelled polygons, to avoid the complication presented in figure 4.)

Solution 1. *[2FV]*

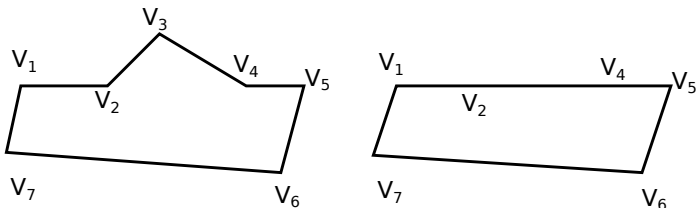Figure 3: Examples of polygons with many sides (odd or even) and only two ears. Figure for [OJN]

Figure 4: A polygon where, removing an ear, the number of unlabelled sides drops from 7 to 4.