Exercises

E8.vii.8 [OKD] Prerequisites: [OKC]. Let X, Y be topological Hausdorff space. Let $E \subseteq X$, let $f : E \to Y$, and suppose that x_0 is an accumulation point of E in X.

- If $\lim_{x \to x_0} f(x) = \ell$ then, for each net $\varphi : J \to X$ with $\lim_{j \in J} \varphi(j) = x_0$ we have $\lim_{j \in J} f(\varphi(j)) = \ell$.
- Consider the filtering set J given by the neighborhoods of x_0; [a] consider nets $\varphi : J \to X$ with the property that $\varphi(U) \in U \setminus \{x_0\}$ for each $U \in J$. We note that $\lim_{j \in J} \varphi(j) = x_0$.

  If for each such net $\lim_{j \in J} f(\varphi(j)) = \ell$, then $\lim_{x \to x_0} f(x) = \ell$.

Solution 1. *[OKF]*

[a] The fact that this is filtering was shown in [O6V], [OGQ] and [OH5]