Exercises

E8.g.8 [0KG] Prerequisites: [0K5], [0KC]. Let X, Y be Hausdorff topological spaces. Let $f : X \to Y$, $x_0 \in X$. The following are equivalent.

1. f is continuous at x_0;
2. for each net $\varphi : J \to X$ such that
$$\lim_{j\in J} \varphi(j) = x_0$$
we have
$$\lim_{j\in J} f(\varphi(j)) = f(x_0) \quad .$$

Hint, for proving that 2 implies 1. Suppose that x_0 is an accumulation point. Consider the filtering set J given by the neighborhoods of x_0; consider nets $\varphi : J \to X$ with the property that $\varphi(U) \in U$ for each $U \in J$; note that $\lim_{j\in J} \varphi(j) = x_0$.

Solution 1. *[0KH]*