

Exercises

E8.86 [OKZ] Prerequisites: [OJ1], [026]. Conversely, let X be a set and \mathcal{B} a family of subsets that verify the previous properties (a),(b) seen in [OKX]. Let σ the family of sets that are obtained as a union of elements of \mathcal{B} , in symbols ^a

$$\sigma \stackrel{\text{def}}{=} \left\{ \bigcup_{i \in I} A_i : I \text{ family of indexes and } A_i \in \mathcal{B} \forall i \in I \right\} ;$$

it is meant that also $\emptyset \in \sigma$. Show that σ is a topology.

Solution 1. [OMO]

^a As already discussed in [026], you could also use the more compact notation $\sigma \stackrel{\text{def}}{=} \left\{ \bigcup \mathcal{F} : \mathcal{F} \subseteq \mathcal{B} \right\}$.