Exercises

E8.viii.11 [OM3]Prerequisites:[OJ1],[OKX],[OKZ]. Let now $X_1, \dots X_n$ be topological spaces with topologies, respectively, $\tau_1, \dots \tau_n$; let $X = \prod_{i=1}^{n} X_i$ be the Cartesian product. We apply the above results to define the **product topology** τ: this can be described in two equivalent ways.

- Union of all Cartesian products of open sets [a]

$$\tau = \left\{ \bigcup_{j \in J} \prod_{i=1}^{n} A_{i,j} \, : A_{1,j} \in \tau_1, \dots A_{n,j} \in \tau_n \forall j \in J, J \right.$$
$$\left. \text{arbitrarily chosen sets of indexes} \right\} .$$

- τ is the smallest topology that contains Cartesian products of open sets.

Solution 1. *[OM4]*

[a] As defined at the beginning of section 6, chapter 5, of the notes [**?**].