

## Exercises

E8.90 [OM5] Prerequisites: [OM3], [OKX], [OKZ]. Let now  $X_1, \dots, X_n$  be topological spaces with topologies  $\tau_1, \dots, \tau_n$  respectively and suppose that  $\mathcal{B}_1, \mathcal{B}_2, \dots, \mathcal{B}_n$  are bases for these spaces. Let  $X = \prod_{i=1}^n X_i$  be the Cartesian product, and let

$$\mathcal{B} = \left\{ \prod_{i=1}^n A_i : A_1 \in \mathcal{B}_1, A_2 \in \mathcal{B}_2, \dots, A_n \in \mathcal{B}_n \right\}$$

The family of all cartesian products of elements chosen from their respective bases. Show that  $\mathcal{B}$  is a base for the product topology. (This exercise generalizes the previous [OM3], taking  $\mathcal{B}_i = \tau_i$ ).

**Solution 1.** [OM6]

See also the exercise [OQM] for an application to the case of metric spaces.