Exercises

E8.74 [OM9] Prerequisites: [O6N], [OKX]. We verify that what is expressed in [OGQ] also applies to the "basis". Let $\mathcal{B}$ be a basis for a topology τ on X; consider the descending order between sets (formally $A \preceq B \iff A \supseteq B$); with this order $(\mathcal{B}, \preceq)$ is a directed set, whose minimum is $\varnothing$. Now suppose the topology is Hausdorff. Then taken $x \in X$, let $\mathcal{U} = \{A \in \mathcal{B} : x \in A\}$ be the family of elements of the base that contain x: show that $\mathcal{U}$ is a directed set. Show that it has minimum if and only if the singleton $\{x\}$ is open.

Solution 1. *[OMB]*