

**Definition 9.2.** [OMS] A **distance** is a function  $d : X \times X \rightarrow [0, \infty)$  that enjoys the following properties:

- $d(x, x) = 0$ ;
- (separation property) if  $d(x, y) = 0$  then  $x = y$ ;
- (symmetry)  $d(x, y) = d(y, x)$  for each  $x, y \in X$ ;
- (triangle inequality)  $d(x, z) \leq d(x, y) + d(y, z)$  for each  $x, y, z \in X$ .