Exercises

E9.4 [OMZ] Given f, g continuous functions on $\mathbb{R}$, we define

$$d(f,g) = \sup_{x\in\mathbb{R}} |f(x) - g(x)| \, .$$

Prove that d is a distance on $X = C(\mathbb{R})$, in the extended sense of the exercise [OMX].

Let $f \sim g \iff d(f,g) < \infty$ as before, show that the family of equivalence classes $X/\sim$ has the cardinality of the continuum.

Solution 1. *[ONO]*