Exercises

E9.4 [ON1] Prerequisites:[OPS].Note:*See also eserc.* [192]. Suppose $\varphi : [0,\infty) \to [0,\infty)$ is monotonic weakly increasing and subadditive, *i.e.* $\varphi(t) + \varphi(s) \geq \varphi(t+s)$ for each $t, s \geq 0$; and suppose that $\varphi(x) = 0$ if and only if $x = 0$.

Then $\varphi \circ d$ is again a distance. Examples: $\varphi(t) = \sqrt{t}$, $\varphi(t) = t/(1+t)$, $\varphi(t) = \arctan(t)$, $\varphi(t) = \min\{t, 1\}$.

Moreover show that if φ is continuous in zero then the associated topology is the same. [a]

Solution 1. [ON2]

[a] See Sec. [2C2] below for a summary of definitions regarding topology in metric spaces: in particular the result [OPS] will be useful.