Exercises

E9.4 [ONC] Let $\varepsilon_n > 0$ be an infinitesimal decreasing sequence. If $(x_n) \subset X$ is a Cauchy sequence, there exists a subsequence n_k such that

$$\forall k \in \mathbb{N},\ \forall h \in \mathbb{N},\ h > k \Rightarrow d(x_{n_k}, x_{n_h}) \leq \varepsilon_k \quad .$$

Solution 1. *[OND]*

This property is often used by choosing $\varepsilon_n = 2^{-n}$, or other sequence whose series converges.