

Exercises

E9.4 [ONM] Prerequisites: [OB9], [192], [ON6]. Difficulty: *. Note: Exercise 2, written exam, 9 July 2011.

Let $\alpha(x)$ be a continuous function on \mathbb{R} , bounded and strictly positive. Given f, g continuous functions on \mathbb{R} , we define

$$d(f, g) = \sup_{x \in \mathbb{R}} (\min\{\alpha(x), |f(x) - g(x)|\}).$$

Prove that d is a distance on $C(\mathbb{R})$ and that $(C(\mathbb{R}), d)$ is complete.

Solution 1. [ONP]